A differential geometry approach for change detection in medical images.

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***Abstract*—Change detection is of paramount importance in medical imaging, serving as a non-invasive quantifiable powerful tool in diagnosis and in assessment of the outcome of treatment of tumors. We present a new quantitative method for detecting changes in volumetric medical data and in clustering of anatomical structures, based on assessment of volumetric distortions that are required in order to deform a test three-dimensional medical dataset segment onto its previously-acquired reference, or a given prototype in the case clustering. Unlike the voxel-based classical techniques of shape comparison, our algorithm operates on tetrahedral meshes and can, therefore be applied on both closed, simply-connected, surfaces and in volumetric domains with more sophisticated boundaries.**

***Keywords*-Brain Imaging; Change detection; Tetrahedral meshes; Volumetric deformations;**

I. Introduction

With the recent advance in 3D medical scanning devices there is a fundamental need for quantifiable geometric properties suitable for disease diagnosis and for detecting changes during therapeutic interventions. As shown by [8], the ability to detect and localize changes in brain tumors, observed by CT or MRI, has a significant implication for clinical manifestations and diagnosis of the disease. Studies suggest that volumetric changes in some brain regions can be attributed to ageing [2], while changes in other areas are linked to certain diseases, such as autism and epilepsy [3].

We present a new geometric approach for detecting changes in medical images and for clustering equivalent classes of anatomical disorders. Our input data are assumed to be a set of volumetric domains, or a set of simply-connected closed-surfaces that enclose well defined interior volumes. In order to detect discrepancy, or assess similarity, between geometric structures, we consider distortions of basic geometric properties associated with deformations of one 3D segment into another.

Our approach is motivated by behavior of conformal and isometric mappings in 3D space, but it can be extended to higher dimensions. In particular, for a deformation function f, defined to be a local diffeomorphism of a volumetric domain, we measure lengthwise and anglewise distortions, referred to as *conformal* and *isometric* distortions, respectively. According to [4] and [5], these quantities are measured at point x as a function of the singular values of the Jacobian matrix of f.

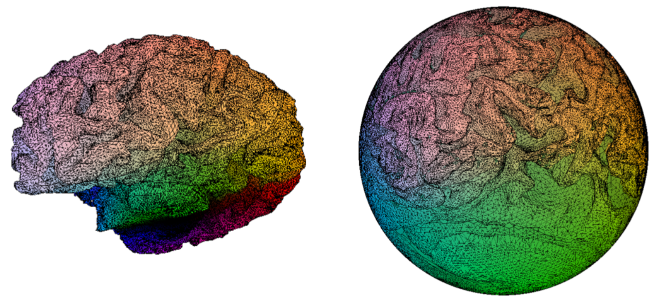

Figure 1. Mapping (1) of a tetrahedral brain model (left) into its bounding ball. RGB color values correspond to spherical coordinates (r,φ,θ) of the source domain.

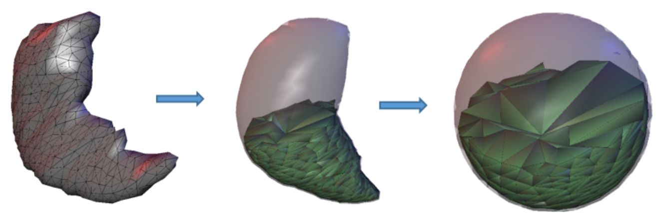

Figure 2. Stages of the proposed algorithm, illustrated for a typical hippocampus model, are included from left to right: original mesh, tetrahedral mesh of a canonical form and its image under deformation (1).

Global changes in geometric structures can be assessed by a weighted average of the distortion measures over a target domain. We adopt the method of [5] which uses partition of discrete volumes into tetrahedrons.

The main step in our algorithms is construction of a deformation function between a given test source and a basic target domain. Since there is no single robust algorithm that arbitrarily deforms one shape into another, we consider deformations of an arbitrary volumetric domain D into a reference ball, defined in spherical coordinates according to [5] by

$$(r,\varphi,\theta) \mapsto \left(R\frac{r}{d(\varphi,\theta)},\varphi,\theta\right), \qquad (1)$$

where $d(\varphi,\theta)$ is a distance from the origin to the farthest point on the boundary, measured at radial angle $\angle(\varphi,\theta)$, and

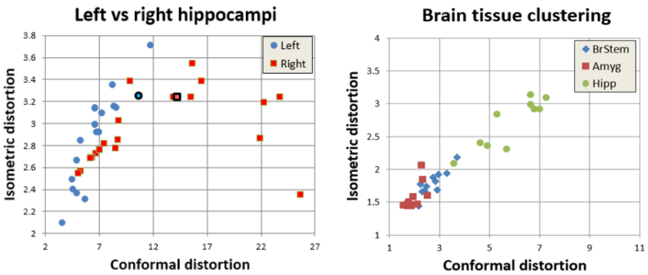

Figure 3. Scatter plots show results of comparison between left and right hyppocampi and classification of brain segments based on Algorithm 1. These segments include: brain stem, amygdala and hippocampus.

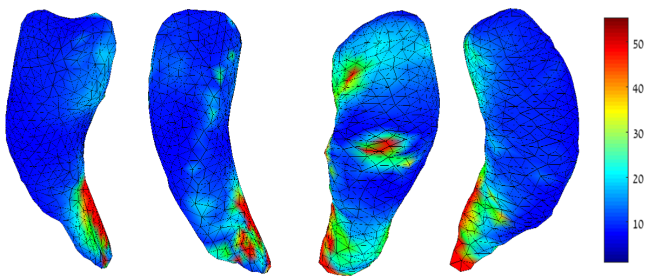

Figure 4. Two left and two right hippocampal regions (shown from left to right). The colors depict conformal distortions measured for volumetric deformation of these regions into a ball defined by (1).

R is a radius of the bounding sphere of the domain D. This method is applicable for discrete representations of medical data, as illustrated in Fig. 1.

In order to deal with artifacts caused by uncertainties associated with scanning devices, we employ a classical multidimensional scaling (MDS) method [9] to construct a canonical form of the input data. This type of preprocessing, removes noise and globally preserves shapes (see Fig. 2).

II. Volumetric distortions

A common way to represent volumetric data for numerical computations is by decomposition of the continuous domain into tetrahedrons by implementing a tetrahedral meshing algorithm. Let (V,T) be a tetrahedral mesh that represents a continuous medical volumetric segment D, where V and T denote vertex and tetrahedra set, respectively. Assume $\boldsymbol{u} = (u_1,\ldots,u_m)$ is the sampling of a given mapping $f : D \to \mathbb{R}^3$ on the vertex set, i.e., $f(v_i) = u_i$ for each $v_i \in V$. Then, the triplet $(V,T,\boldsymbol{u})$ defines a discrete deformation of the volume D.

Algorithm 1 measures conformal and isometric distortions associated with the volumetric mapping $(V,T,\boldsymbol{u})$ via construction of simplical mappings (for details see [5]). In particular, if the mapping f is defined according to (1), then a weighted average of the resultant distortions over source tetrahedra (Fig. 3) provides a qualitative measure of the similarity between the segment D and the bounding ball. While distortion measures per boundary vertex (Fig. 4) indicate the local deviation from a round surface, or from the bounding surface of another suitable geometrical reference object of choice such as, for example, ellipsoid, cylinder or torus (see Fig. 5).

Weights used in the averaging of volumetric distortions can be directly sampled from the values of MRI intensity, or the proper signal characterizing the physical measure of any other imaging modality.

Algorithm 1: ComputeDistortions$(V,T,\boldsymbol{u})$

Input:

Deformation of a tetrahedral mesh (V,T) represented by $\boldsymbol{u}$.

- Construct the simplical mapping $\phi : \bigcup_{t\in T}\{t\} \to \mathbb{R}^3$ such that $\phi(v_i) = u_i$ for each $v_i \in V$.

foreach $t \in T$ **do**

- $\phi_t \hookleftarrow$ linear part of ϕ restricted to t.
- $(\sigma_1,\sigma_2,\sigma_3) \hookleftarrow$ singular values of ϕ_t put in the descending order.
- $\mathrm{conf}(t) \hookleftarrow \max\left\{\dfrac{\sigma_1^2}{\sigma_2\sigma_3}, \dfrac{\sigma_1\sigma_2}{\sigma_3^2}\right\}$.
- $\mathrm{isom}(t) \hookleftarrow \max\left\{\sigma_1, \sigma_3^{-1}\right\}$.

Output:

Conformal and isometric distortions per each tetrahedron $t \in T$, denoted by $\mathrm{conf}(t)$ and $\mathrm{isom}(t)$, respectively.

III. Implementation

We illustrate the implementation of our geometric approach in sorting of brain tissue obtained by our colleagues at the Dalian Medical University. The first database (Fig. 3) contains segments of 44 brain CT scans that include healthy subjects and epileptic patients. The second dataset contains brain tissue scans of additional 200 subjects (Fig. 6). The boundary of each segment is approximated by a triangular surface and the enclosed volume is represented by a tetrahedral mesh produced via Delaunay triangulation.

As shown in Fig. 3 and 6, our method can classify various segments of brain scans and distinguish between left and right counterparts of hippocampi. The approach is applicable also to other organs and can be employed in statistical analysis.

Furthermore, Algorithm 1 can be straightforwardly generalized to deal with wide class of geometric measures and energies, locally expressed by the singular values $\sigma_1,\sigma_2,\sigma_3$. For instance, the elasticity and the smoothness energies, measured over source tetrahedron $t \in T$, can be estimated by the quantities $(\sigma_1 - 1)^2 + (\sigma_3 - 1)^2$ and $\sigma_1^2 + \sigma_2^2 + \sigma_3^2$, respectively.

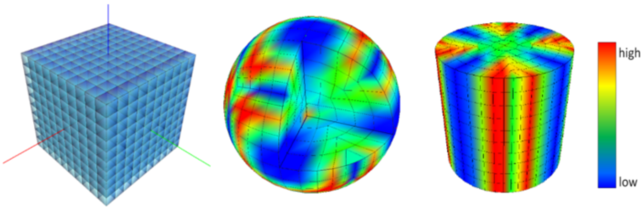

Figure 5. The figure contains (from left to right) : source cube, cross section of its image under the mapping (1), deformation of the cube into a solid cylinder. The last deformation is constructed according to [5]. Colors of target domains depict distribution of conformal distortion.

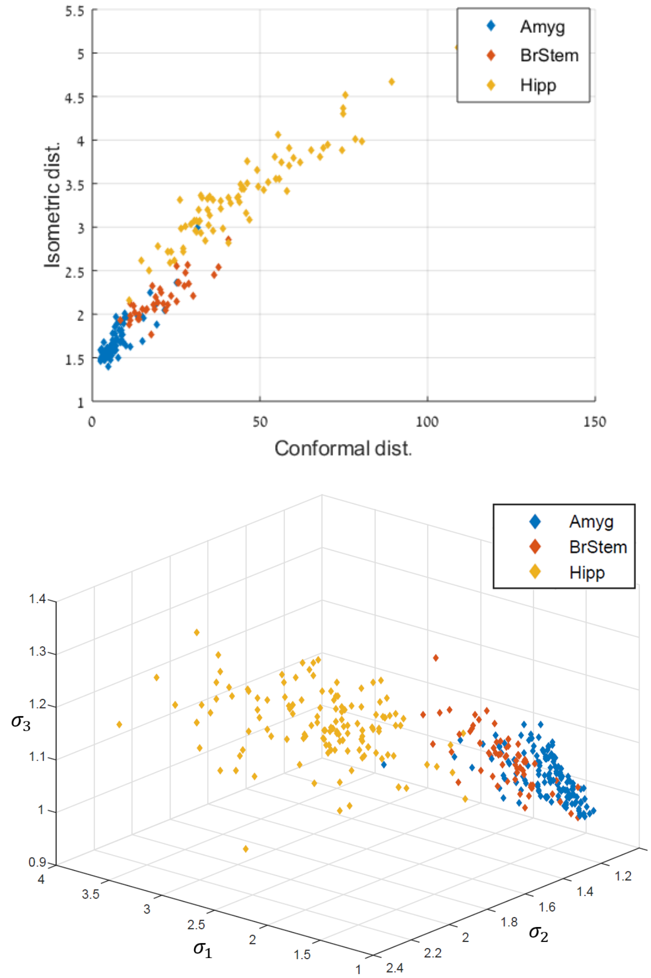

Figure 6. Scatter plots of segments obtained from brain stem, amygdala and hippocampus. The data representations are based on quantifying conformal and isometric distortions (top) and on the comparison of average singular values $(\sigma_1, \sigma_2, \sigma_3)$, obtained by Algorithm 1 (bottom).

IV. Conclusion

Compared with classical methods based on voxel morphology [1], [2] and related studies [7], our technique has the merit of both the global and individual comparisons. Moreover, tetrahedral meshes require in general much lower number of data samples than voxel images, since it constitutes a 3D signal-specific nonuniform representation scheme.

Our algorithm is capable of detecting small deviations in curvature and smoothness of the boundary surface. According to some cancer studies [8], these deviations play a critical rote in decoding tumor phenotype. On the other hand, our technique is likely to rely more on the quality of boundary surfaces obtained in the segmentation process.

Since we employ pure geometric tools, our algorithm is fast and it requires much simpler implementation than related dictionary learning techniques [6], [7].

Medical validations and human inspections will be included in future work.

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