# Multi-Resolution Approach to Computing Locally Injective Maps on Meshes

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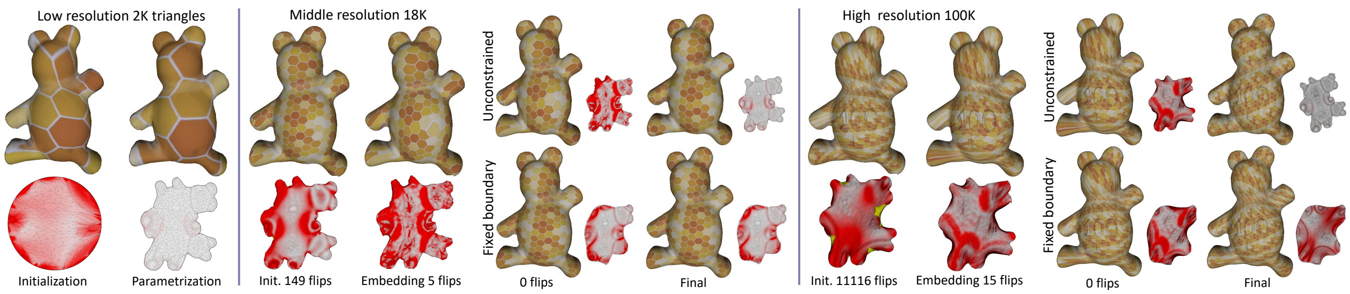


**Figure 1: Hierarchical surface parametrization within three resolution levels.** The lowest resolution is parametrized by minimizing the isometric distortion via BQCN solver with the standard initialization. Four stages of unconstrained (top) and constrained (bottom) parametrizations are illustrated for the middle and hight resolutions. These stages include (from left to right): Tutte embedding into the boundary of a decimated parametrization, an optimized Tutte embedding, the point where all flipped triangles (shown in yellow) are repaired by MBCD algorithm, and the final parametrization results. In the constrained parametrization, the boundary of an optimized Tutte embedding is fixed (the bottom of columns 5,6,9,10).

## CCS CONCEPTS

• **Computing methodologies** → *Shape modeling*.

## KEYWORDS

Parameterizations, low distortion, foldover-free, shape deformation.

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## 1 INTRODUCTION

Computing injective mappings with low distortions on meshes is an important problem for its wide range of practical applications in computer graphics, geometric modeling and physical simulations. Such tasks as surface parametrization or shape deformation are often reduced to minimizing non-convex and non-linear geometric energies defined over triangulated domains. These energies are commonly expressed in a finite element manner as a weighted sum of distortion densities $\mathcal{D}$ over simplixes $\mathcal{S}$:

$$E\left(f[\boldsymbol{x}]\right) = \sum_{s\in\mathcal{S}} w(s)\mathcal{D}(J_s)\,, \tag{1}$$


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where $\boldsymbol{x}$ is the column stack of vertex positions under piecewise affine mapping $f$ and $J_s$ denotes the Jacobian of $f$ on $s$, modulo a rigid transformation of $s$ to its target shape. Usually, a proper minimizer of (1) has to satisfy the following constraints:

$$\det(J_s) \quad > 0,\quad \forall s\in\mathcal{S}; \tag{2}$$

$$A\boldsymbol{x} \quad = \boldsymbol{b}\,, \tag{3}$$

where (2) enforces $f$ to preserve orientation of each simplex, and $(A, \boldsymbol{b})$ is a linear system of the given positional constraints. The orientation constraints are particularly important in parametrization problems, since they avoid undesirable foldover artifacts in the texture, while positional constraints are widely used in shape deformation applications, such as point-to-point deformations, deformations with fixed anchors, and more.

In this work we propose a multi-resolution approach to construction of injective maps with low distortions in 2D and 3D, by initializing the optimization of (1) with the solution of the same problem in a lower resolution. In certain aspects our approach extends the recently proposed Progressive Parametrization [Liu et al. 2018], by decomposing the objective map $f$ into number of intermediate mappings with decreasing distortions and diminishing number of simplexes.

Although our approach is simple and intuitive, it has not been yet properly implemented on meshes due to the following major limitations of state-of-the-art methods: **i)** fast optimizers of (1) have to be initialized by an injective map $f^0$ [Claici et al. 2017; Rabinovich et al. 2017; Shtengel et al. 2017; Zhu et al. 2018]; **ii)** solvers that can handle non-injective initializations are either slow and hardly scalable [Kovalsky et al. 2014], or require a user assistance (non-automatic) [Poranne et al. 2017]; **iii)** methods for computing injective maps are effective only in simple configurations [Aigerman and Lipman 2013; Fu and Liu 2016; Kovalsky et al. 2015].

Despite long history of multi-resolution algorithms in computer graphics, dating back to late 90s [Lee et al. 1998], the vast majority of these methods operate over multigrid domains, spline-based models and subdivision surfaces. Due to the aforementioned difficulties in processing non-locally injective maps multi-resolution methods have surprisingly failed, so far, to perform competitively on meshes. In our method, the non-injective initialization obstacle, often occurring during transitions between multiple resolutions, is overcome by simultaneously repairing inverted elements and minimizing distortions by means of a novel Multi-Objective Block Coordinate Descent (MBCD) algorithm [Naitsat and Zeevi 2019]. MBCD is an adaptive local-global solver that alternates between distortion energies (1) and inverted element penalties which are optimized in coordinate blocks of varying sizes.

## 2 OUR APPROACH

Our algorithm reduces the task of minimizing (1) into a number of simplified problems induced over decimated meshes. As illustrated by Figure 2, it contains the following major steps:

(1) Decompose the source mesh $M_0 = (\mathcal{S}_0, \mathcal{V}_0)$ into nested meshes $M_k, M_{k-1}, ..., M_0$; $\forall i$: $M_i = (\mathcal{S}_i, \mathcal{V}_i)$, $|\mathcal{V}_i| < |\mathcal{V}_{i-1}|$.
(2) Using standard methods, compute the mapping of $M_k$: $f_k[\boldsymbol{x}_k] = \text{argmin}_{M_k} E(f)$. Set $i \longleftarrow k-1$.
(3) Compute the initialization $f_i^0$ in the higher resolution using positions of $\mathcal{V}_{i+1}$ vertices under $f_{i+1}$.
(4) Solve $f_i[\boldsymbol{x}_i] = \text{argmin}_{M_i} E(f)$, initialized by $f_i^0$.
(5) If $i > 0$ set $i \longleftarrow i-1$ and repeat step 3.

The exact implementation of step 3 depends on the particular application of our method. In parameterizing disc-topology surfaces, $f_i^0$ is a modified Tutte embedding of $(\mathcal{S}_i, \mathcal{V}_i)$ aimed at placing $\partial\mathcal{V}_i$ on the polygon of $f_i(\partial\mathcal{V}_{i+1})$ target vertices. We introduce two methods for reducing number of initial foldovers. In the first method, we adaptively smooth the boundary polygon and optimize the initial embedding of interior vertices using a modified version of the method presented in [Xu et al. 2011]. Second method initializes $f_i$ by $g_{i+1}(\boldsymbol{x}_i^*)$, where $\boldsymbol{x}_i^*$ and $\boldsymbol{x}_{i+1}^*$ are Tutte maps of meshes in the successive resolutions, and $g_{i+1}$ is the simplicial map that deforms $\boldsymbol{x}_{i+1}^*$ to $\boldsymbol{x}_{i+1}$. Similarly, in shape deformation applications, we set $f_i^0 = f_{i+1}$ on $\mathcal{V}_{i+1}$ and $f_i^0(v)$ on $\mathcal{V}_i \setminus \mathcal{V}_{i+1}$ is computed by interpolating $f_{i+1}(u)$ coordinates over neighbors $u \in \mathcal{V}_{i+1}$ of $v$.

The standard approach to free boundary parametrization is to set $f^0$ to be a Tutte mapping onto a convex planar domain [Floater 2003], such as a disc. These initializations satisfy (2), and thus are compatible with state-of-the-art geometric solvers. However, mapping complex shapes to disc-like domains yields a huge isometric distortion that often costs hundreds of additional iterations for a typical solver. Motivated by the above observations, we suggest that mapping to other domains, similar to source mesh structure, is a much better starting point if the algorithm can effectively fix occasional foldovers. Consequently, we obtain the initial target domain by parameterizing a decimated shape, while MBCD algorithm is employed for repairing foldovers that may result in embeddings onto non-convex domains. After all inverted triangles are fixed, MBCD converges to the BQCN solver [Zhu et al. 2018].

A similar strategy is employed in shape deformation problems. Since anchor points are not necessarily placed on the boundary, we compute the initial deformation as a function of the entire shape obtained by solving a decimated problem.

Our parametrization method was tested on the cut mesh dataset [Liu et al. 2018]. Figures 1 and 2 depict examples of constrained and unconstrained parametrizations with three resolution levels. Some results of multi-resolution shape deformations in 3D are demonstrated in the supplementary material.

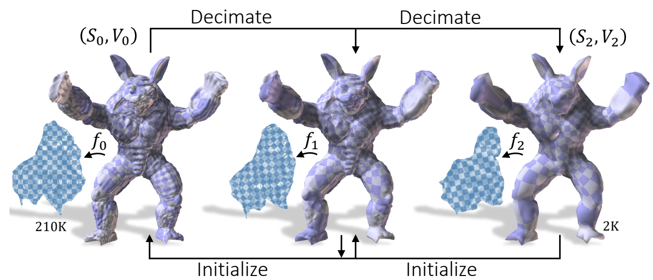


**Figure 2:** Multi-resolution scheme, illustrated for conformal surface parametrization with hierarchies of 210K, 21K and 2K triangles.

## ACKNOWLEDGMENTS

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## REFERENCES

Noam Aigerman and Yaron Lipman. 2013. Injective and bounded distortion mappings in 3D. *ACM Trans. Graph* 32, 4 (2013), 106:1–106:14.

S Claici, M Bessmeltsev, S Schaefer, and J Solomon. 2017. Isometry-Aware Preconditioning for Mesh Parameterization. In *Computer Graphics Forum*, Vol. 36. Wiley Online Library, 37–47.

Michael Floater. 2003. One-to-one piecewise linear mappings over triangulations. *Math. Comp.* 72, 242 (2003), 685–696.

Xiao-Ming Fu and Yang Liu. 2016. Computing inversion-free mappings by simplex assembly. *ACM Transactions on Graphics (TOG)* 35, 6 (2016), 216.

Shahar Z Kovalsky, Noam Aigerman, Ronen Basri, and Yaron Lipman. 2014. Controlling Singular Values with Semidefinite Programming. *ACM Transactions on Graphics (TOG)* 33, 4 (2014), 68.

Shahar Z Kovalsky, Noam Aigerman, Ronen Basri, and Yaron Lipman. 2015. Large-scale bounded distortion mappings. *ACM Trans. Graph* 34, 6 (2015), 191.

Aaron WF Lee, Wim Sweldens, Peter Schröder, Lawrence C Cowsar, and David P Dobkin. 1998. MAPS: Multiresolution adaptive parameterization of surfaces. In *Siggraph*, Vol. 98. 95–104.

Ligang Liu, Chunyang Ye, Ruiqi Ni, and Xiao-Ming Fu. 2018. Progressive parameterizations. *ACM Transactions on Graphics (TOG)* 37, 4 (2018), 41.

Alexander Naitsat and Y. Yehoshua Zeevi. 2019. Adaptive block coordinate descent for distortion minimization. In *Proceedings of the Symposium on Geometry Processing, to appear*. Eurographics Association.

Roi Poranne, Marco Tarini, Sandro Huber, Daniele Panozzo, and Olga Sorkine-Hornung. 2017. Autocuts: simultaneous distortion and cut optimization for UV mapping. *ACM Transactions on Graphics (TOG)* 36, 6 (2017), 215.

Michael Rabinovich, Roi Poranne, Daniele Panozzo, and Olga Sorkine-Hornung. 2017. Scalable locally injective mappings. *ACM Transactions on Graphics (TOG)* 36, 2 (2017), 16.

Anna Shtengel, Roi Poranne, Olga Sorkine-Hornung, Shahar Z Kovalsky, and Yaron Lipman. 2017. Geometric optimization via composite majorization. *ACM Trans. Graph* 36, 4 (2017), 38.

Yin Xu, Renjie Chen, Craig Gotsman, and Ligang Liu. 2011. Embedding a triangular graph within a given boundary. *Computer Aided Geometric Design* 28, 6 (2011), 349–356.

Yufeng Zhu, Robert Bridson, and Danny M Kaufman. 2018. Blended cured quasi-newton for distortion optimization. *ACM Transactions on Graphics (TOG)* 37, 4 (2018), 40.