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Multi-Resolution Approach to Computing Locally Injective Maps on Meshes

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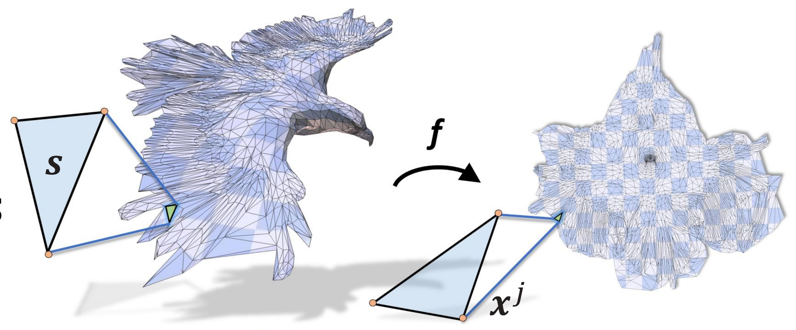
Problem

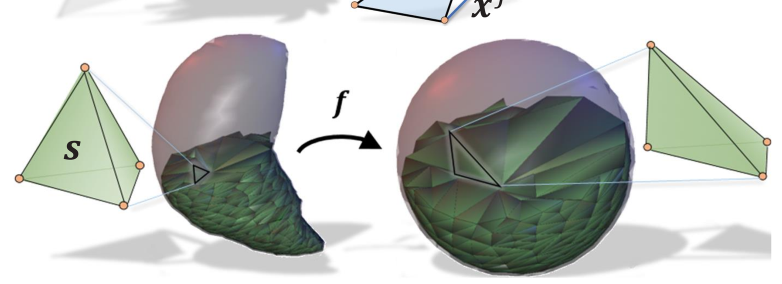
- How to utilize low resolution data for computing locally injective maps with minimal geometric distortions on meshes.

$E\left(f[\boldsymbol{x}]\right) = \sum_{s\in S} w(s)\mathcal{D}(J_s)$, $M = (S, V)$, S-simplexes, V-vertices.

2D: triangle meshes
- Parametrizations
- Planar deformations

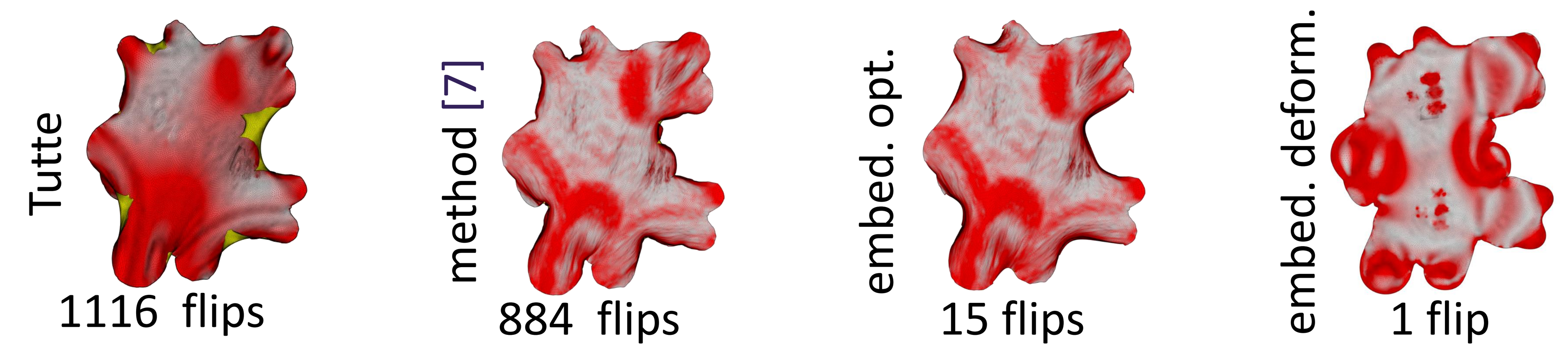


3D: tetrahedral meshes
- Shape deformations
- Simulations



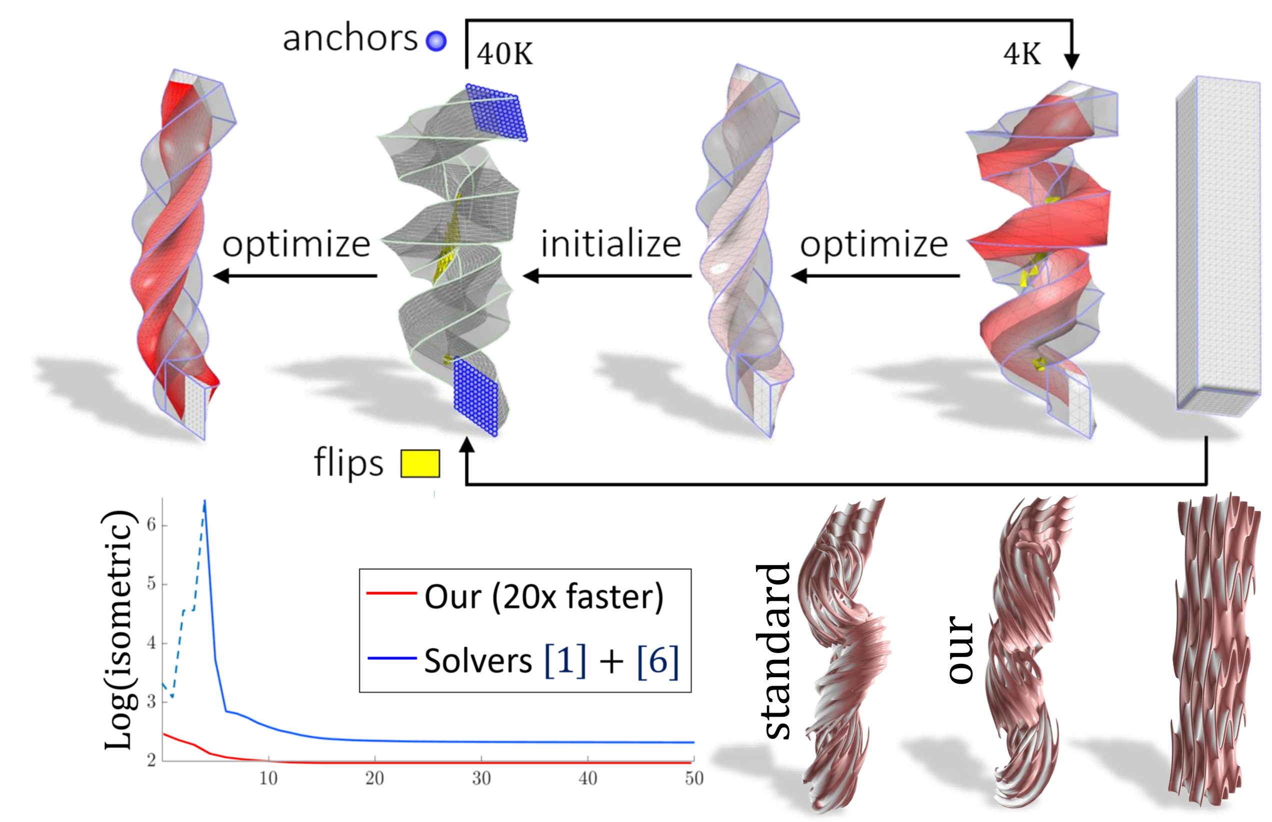
Embedding onto nonconvex domains

- **Embedding optimization**
  1. $\boldsymbol{x}_i^0 \leftarrow$ mapping of M_i onto given boundary region $\boldsymbol{b}$ by [7].
  2. $U \leftarrow$ boundary vertices of flipped triangles; return $\boldsymbol{x}_i^0$ if $U = \emptyset$.
  3. Smooth $\boldsymbol{b}$, $\forall u \in U$; repeat (1).
- **Embedding deformation**
  1. $\boldsymbol{x}_i^*$, $\boldsymbol{x}_{i+1}^* \leftarrow$ Tutte maps of M_i, M_{i+1}.
  2. $g_{i+1} \leftarrow$ simplicial map of $\boldsymbol{x}_{i+1}^*$ onto parametrization $\boldsymbol{x}_{i+1}$.
  3. Return $\boldsymbol{x}_i^0 = g_{i+1}\left((1-\epsilon)\boldsymbol{x}_i^*\right)$



Shape deformation

- Apply embedding deformation with $\boldsymbol{x}_i^*$, $\boldsymbol{x}_{i+1}^*$ = source coordinates of V_i, V_{i+1}.

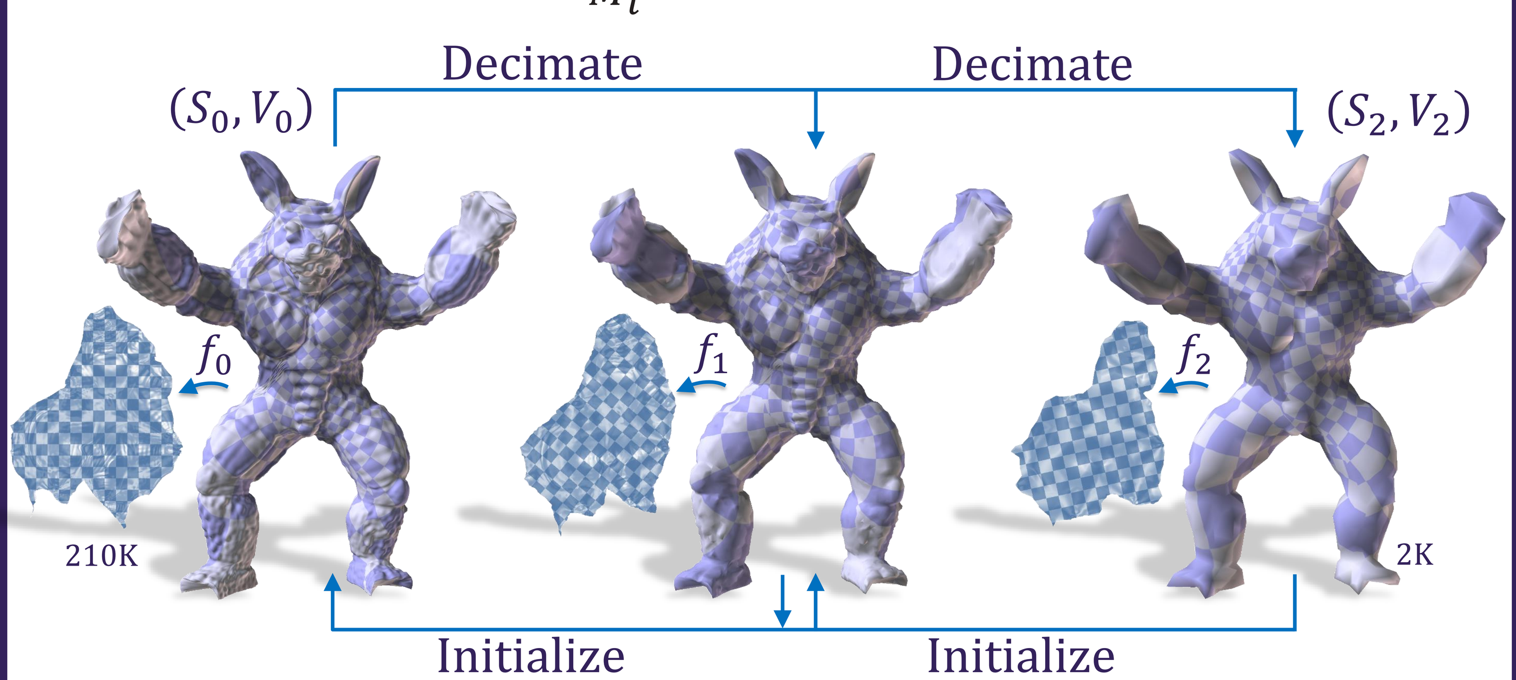


Our approach

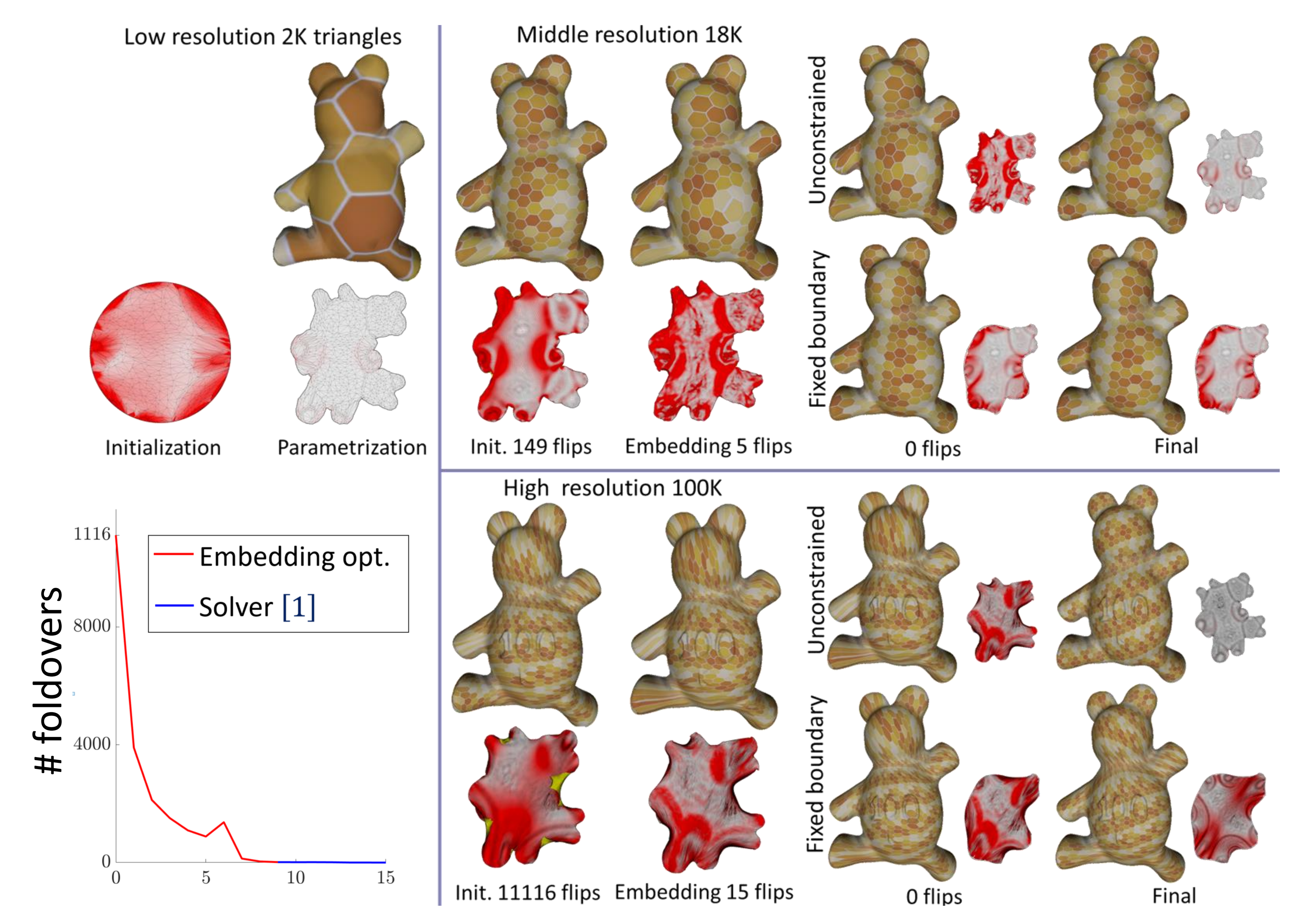
- Initialize $f[\boldsymbol{x}]$ using the solution of the same problem in a lower resolution.
- Fix non-injective initializations by the proposed embedding methods and by [1].

Multi-resolution scheme

1. Decompose M_0 into nested meshes
   - $M_k, M_{k-1}, \dots, M_0$; $M_i = (S_i, V_i)$, $|V_i| < |V_{i-1}|$
2. Compute map of M_k by standard methods
   - $f_k[\boldsymbol{x}_k] = \min_{M_k} E(f)$, f_k^0 = Tutte map
3. For $i = k-1$ to 0
   I. Compute f_i^0 using f_{i+1} mapping
   II. Solve $f_i = \min_{M_i} E(f)$, initialized by f_i^0



Parametrization results



Applications

- Fast hierarchical parametrization of meshes in high resolution.
- Texture transfer from low to high resolution models using constrained version of our method.
- Acceleration of shape deformations and physical simulations.

Future work

- Integration with algorithms for globally injective mapping [8].
- Employing methods [2] and [5] for embedding onto nonconvex boundaries.
- Integration with the OptCuts algorithm [4] for accelerating global parametrization.
- Accelerating optimizations in our method by [3].

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